Free Fall Motion
Introduction to Projectile Motion

Physics

Assumptions

- The upward direction is taken as the positive direction. If the origin is taken at ground level, then:
  - **yi :** is the initial height of the object at time zero.
  - **vi :** is the initial velocity of the object in the y-direction.

Free Fall

- When an object is propelled into the air, it is assumed that all other forces acting on the object except gravity are negligible. This means that:
  - We neglect any effects due to air resistance on the object.
  - We neglect any effects due to the Earth's rotation.
  - We assume that the object does not rise high enough for the acceleration of gravity to change .
- With these assumptions the body's acceleration is both constant and downward regardless of its direction of motion or its height above the ground. This means that object's acceleration is downwards regardless of weather the object is moving upwards or downwards.
- **a = -g = -**9.80 m/s2 .

Frame of Reference – Downward Negative

- The freefall equations were derived by assuming the upward **y-**direction is the positive direction and the clock starts at time **t = 0**. The sign of the acceleration is negative and it equal to **a = -g .** (When an object is moving downwards, its velocity will be a negative number using this frame of reference.)

Frame of Reference – Downward Positive

- It is possible and sometimes useful to take the downward direction as positive. **In this case the equations of motion will not be identical to those above**. Typically one would use this frame if the object's motion was all downwards like a ball thrown downwards from the top of a building. The main difference between these two frames is the sign of the velocity and acceleration.

Galileo Galilei and The Law of Falling Bodies:

- **In the absence of air resistance, any two bodies that are dropped from rest at the same moment will reach the ground at the same time regardless of their mass.**
- A stronger statement is: The acceleration of all objects is the same in the absence of air resistance. As long as an object is in freefall - regardless if it is going up, down or sideways - its acceleration is equal to

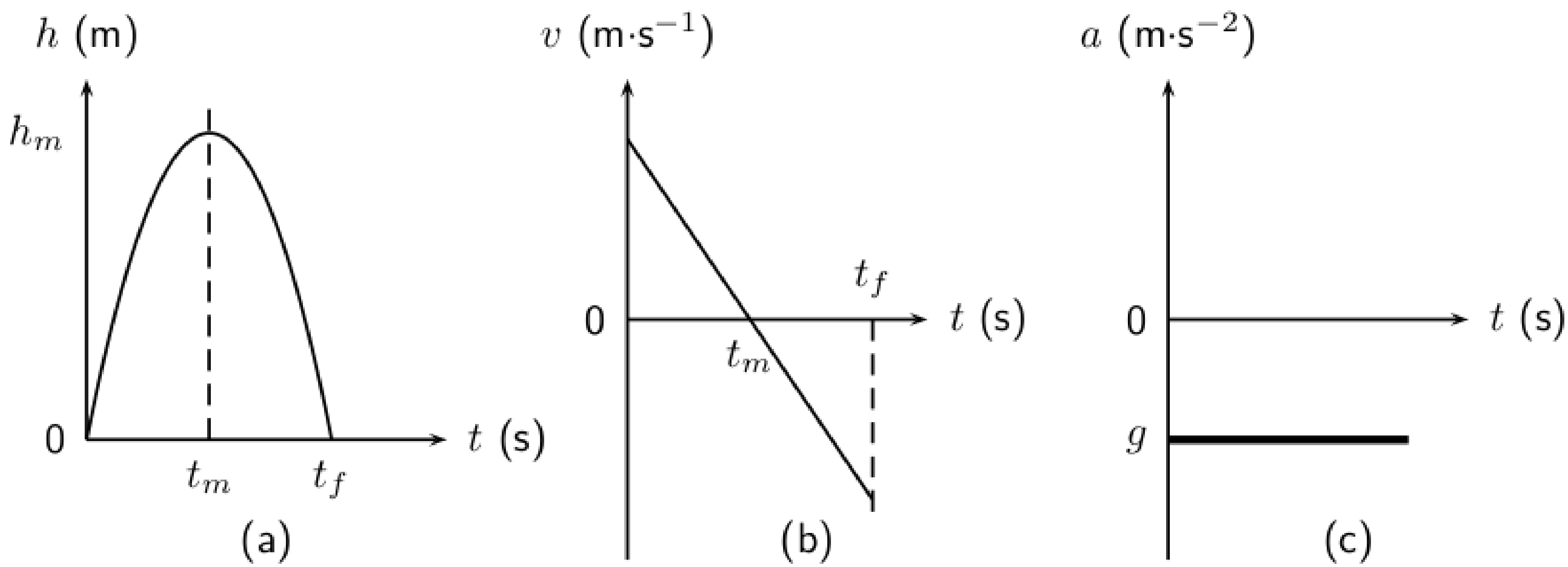
$9.80\ m/s^2$ downwards.

The Law of Falling Bodies:

- Two identical objects are dropped from the same height at the same time. With air resistance set to zero, both balls strike the ground at the same time. As the air resistance is increased, the more massive object will strike the ground first. With enough air resistance the lighter ball reaches terminal velocity.

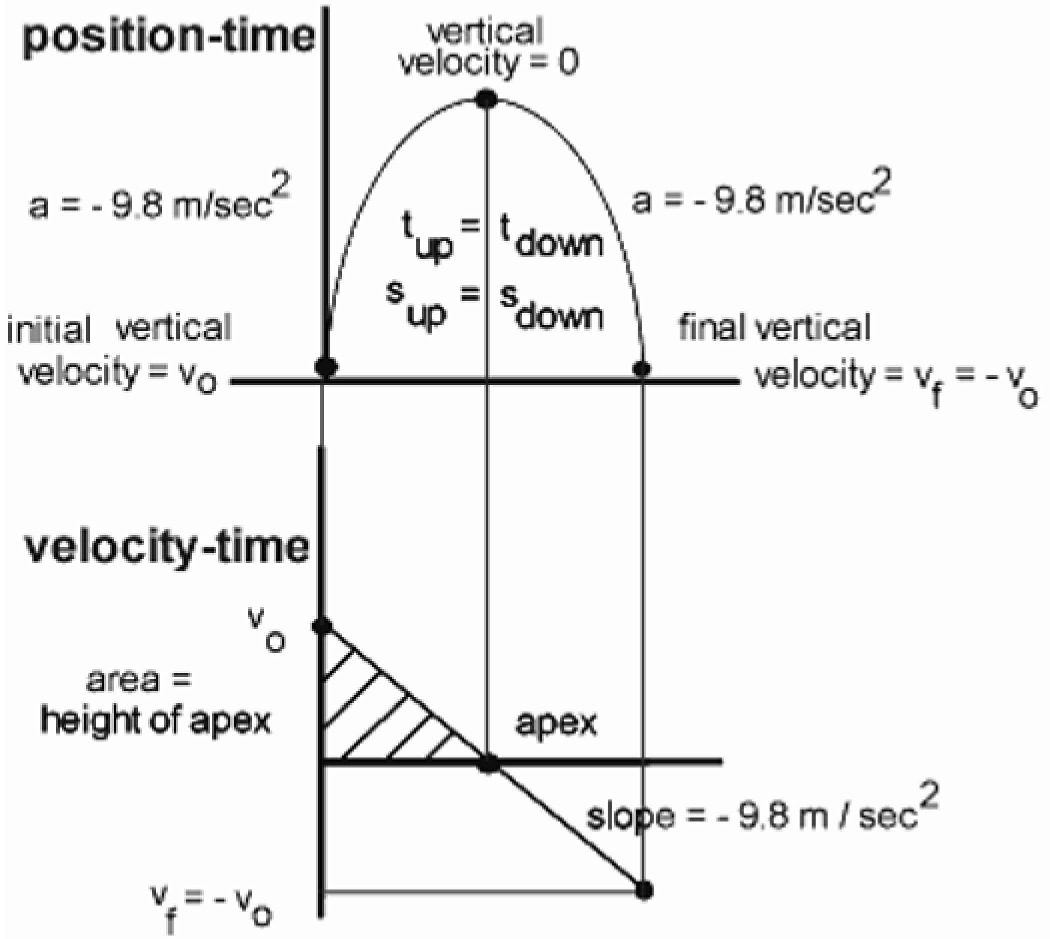
Graphing Free Fall Motion.

distance vs. time, velocity vs. time, and the acceleration vs. time diagrams.



g = - 9.8 m/s^2

Graph Explanation



Derivation of the Free-Fall Equations:

Since we usually associate the vertical direction with the y-axis, we will transform the generic equations for constant acceleration by first replacing " **x** " with " **y** ", " **a** " with "**-g**", and " v " with " vy ".**

$$a \rightarrow -g$$

Then,

$$x \rightarrow y \qquad x_o \rightarrow y_o$$

$$v \rightarrow v_y \qquad v_o \rightarrow v_{y,o}$$

$$v = a\,t + v_o \qquad \rightarrow \qquad v_y = -g\,t + v_{y,o}$$

$$\bar{v} = \frac{1}{2}(v + v_o) \qquad \rightarrow \qquad \bar{v}_y = \frac{1}{2}(v_y + v_{y,o})$$

$$x = \frac{1}{2}a\,t^2 + v_o\,t + x_o \qquad \rightarrow \qquad y = -\frac{1}{2}g\,t^2 + v_{y,o}\,t + y_o$$

$$x = \bar{v}\,t + x_o \qquad \rightarrow \qquad y = \bar{v}_y\,t + y_o$$

$$x = \frac{v^2 - v_o^2}{2a} + x_o \qquad \rightarrow \qquad y = \frac{v_y^2 - v_{y,o}^2}{-2g} + y_o$$

Free fall equations-Simplified

- We use kinematics equations because as Galileo described free-fall motion is a constant acceleration.
- Although all equations are used the most prevalent equation is #1 and #3 or #5(depending on your direction) where a=-g.
- The following are rearrangements of #1 and #3 or #5 to find distance traveled, time and velocity at a given point of fall.
- Initial velocity is usually made to be zero.

1) $v_f = v_i + at$

2) $v_f^2 = v_i^2 + 2a\Delta x$

3) $\Delta x = v_i t + \frac{1}{2}at^2$

4) $\Delta x = \frac{1}{2}(v_f + v_i)t$

5) $\Delta x = v_f t - \frac{1}{2}at^2$

Kinematics Equations

Free Fall Equation: Find time given height

$$t = \sqrt{\frac{2\Delta x}{g}}$$

Δx = height

$g = 9.8\ m/s^2$

Finding Height with Time

$$\Delta x = \frac{gt^2}{2}$$

vi: initial velocity=0

Δx = height

$g = 9.8\ m/s^2$

Calculating fall velocity

$$vf = gt$$

vf: final velocity

vi: initial velocity=0

$g = 9.8\ m/s^2$

vf of falling projectile = vi of launch

Up or Down

'...an object is dropped'

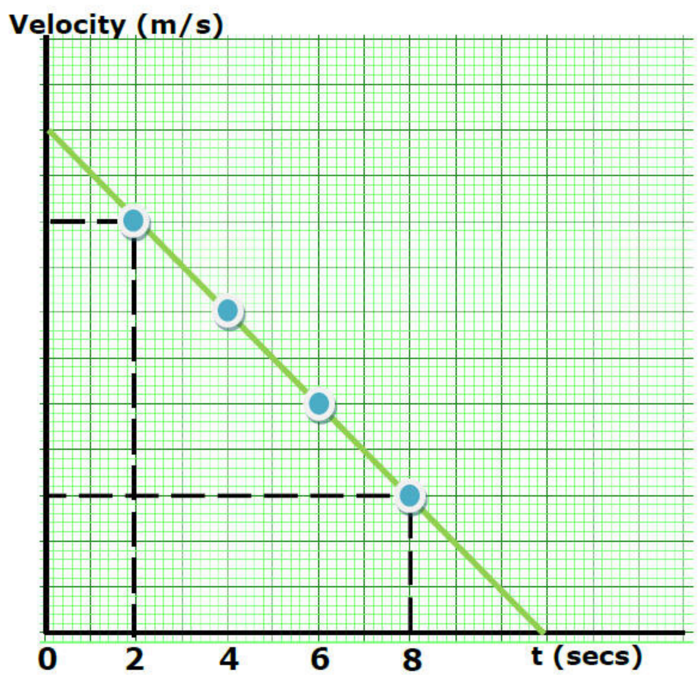
Vi = 0

a = g = +9.8m/s2

(make down +)

... an object is ascending:

a = g = -9.8 m/s2 (make down -)



Ex) Object thrown up Vi = 110 m/s, sketch a V vs t plot of the object from the throw to the peak (use a = -10 m/s2)

Example Problem

A bag is dropped from a hovering helicopter. When the bag has fallen 2.0 s,

a. what is the bag's velocity?

$$v = v_0 + at$$
$$= 0.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(2.0 \text{ s})$$
$$= -2.0 \times 10^1 \text{ m/s}$$

b. how far has the bag fallen?

$$d = v_0 t + \frac{1}{2} a t^2$$
$$= 0.0 \text{ m/s} + \frac{1}{2}(-9.80 \text{ m/s}^2)(2.0 \text{ s})^2$$
$$= -2.0 \times 10^1 \text{ m}$$

The bag has fallen 2.0×10^1 m.

Example Problem 2

A weather balloon is floating at a constant height above Earth when it releases a pack of instruments.

a. If the pack hits the ground with a velocity of –73.5 m/s, how far did the pack fall?

$$v^2 = v_0^2 + 2ad$$

$$d = \frac{v^2 - v_0^2}{2g}$$

$$= \frac{(-73.5\ \text{m/s})^2 - (0.00\ \text{m/s})^2}{(2)(-9.80\ \text{m/s}^2)}$$

$$= \frac{5402\ \text{m}^2/\text{s}^2}{-19.6\ \text{m/s}^2} = -276\ \text{m}$$

The pack fell 276 m.

b. How long did it take for the pack to fall?

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a}$$

$$= \frac{-73.5\ \text{m/s} - 0.00\ \text{m/s}}{-9.80\ \text{m/s}^2}$$

$$= 7.50\ \text{s}$$

Example Problem 3

During a baseball game, a batter hits a high pop-up. If the ball remains in the air for 6.0 s, how high does it rise? **Hint:** Calculate the height using the second half of the trajectory.

The time to fall is 3.0 s

$$d = vt + \frac{1}{2}at^2$$

$$= 0.0\text{ m} + \frac{1}{2}(-9.8\text{ m/s}^2)(3.0\text{ s})^2$$

$$= -44\text{ m}$$

The ball rises 44 m, the same distance it falls.

Example Problem 4

An astronaut drops a feather from 1.2 m above the surface of the moon. If the acceleration of gravity on the moon is 1.62 m/s^2 downward, how long does it take the feather to hit the moon's surface?

$$d = v_0 t + \frac{1}{2} a t^2$$

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{(2)(-1.2 \text{ m})}{(1.62 \text{ m/s}^2)}} = 1.2 \text{ s}$$

Example Problem 5

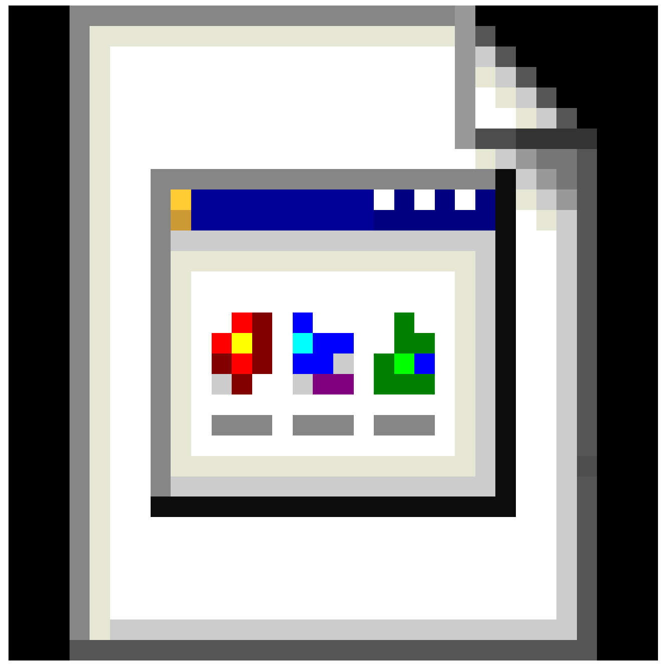
A stone falls freely from rest for 8.0 s.

a. Calculate the stone's velocity after 8.0 s.

$$v = v_0 + at$$
$$= 0.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(8.0 \text{ s})$$
$$= -78 \text{ m/s (downward)}$$

b. What is the stone's displacement during this time?

$$d = v_0 t + \frac{1}{2} a t^2$$
$$= 0.0 \text{ m} + \frac{1}{2}(-9.80 \text{ m/s}^2)(8.0 \text{ s})^2$$
$$= -3.1 \times 10^2 \text{ m}$$



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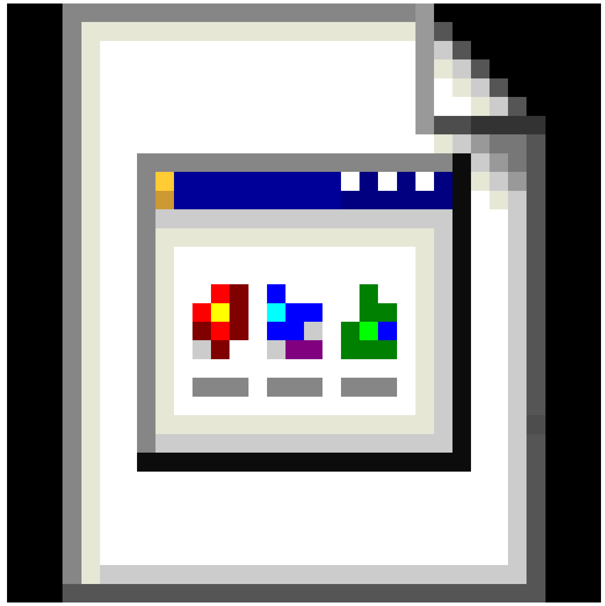

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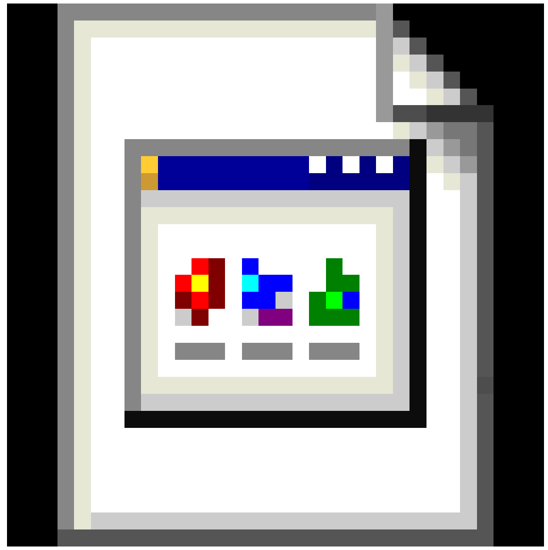

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